question can be resolved by comparing the required vertical acceleration of the collectors [found by differentiating Eq. (6)] to the vertical acceleration that is caused by gravity gradient forces. The first is easily found to be $a_c = \dot{V}_c = -K_{CS}(l/2)\omega_0^2 \sin\phi \cos(\omega_0 t)$. The gravity gradient acceleration at the ends of a two-mass dumbell with length l is given by $a_{gg} = 3l\omega_0^2$, so that the ratio (a_{gg}/a_C) is

$$(a_{gg}/a_C) = 6/[K_{CS} \sin\phi \cos(\omega_0 t)] \tag{15}$$

Since $K_{CS} < 1$, it is clear that the gravity gradient force will always be at least six times the minimum acceptable value and will usually be much greater than that.

Conclusions

The analysis above shows that the thrust magnitude and the fuel requirements necessary for a hanging tether interferometer with a 10-km baseline are well within the present "state of the art" for a system operating in synchronous orbit, but are questionable in low Earth orbit. The thrusting necessary to oppose Coriolis forces would probably be provided by ion thrusters operating continuously using the variable thrust program also derived above. The major unanswered questions affecting the practicality of the system concern vibrations induced in the tether by the CS crawling mechanism, the reeling mechanism, and perhaps by the ion thrusters. The magnitude of these vibrations and their effect on the optical system and the attitude-control system needs to be examined both analytically and experimentally. The analytical investigation is progressing and hopefully the experimental investigation will soon follow.

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Krylov Model Reduction Algorithm for Undamped Structural Dynamics Systems

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Introduction

KRYLOV vectors have been shown to form an efficient basis for eigenvalue analysis and model reduction of structural dynamics systems.[1-3] In Ref. 4, a Krylov model reduction algorithm is developed for a damped structural dynamics system described by a matrix second-order differential equation together with an output measurement equation. Numerical results have shown that Krylov-based reduced models have some advantages over normal mode reduced models in the application to structural control problems. The purpose of this Note is to present a Krylov model reduction algorithm for undamped systems. The undamped case covered in this Note supplements the damped case in Ref. 4, completing the work on Krylov model reduction methods.

The Krylov model reduction algorithm presented here has a feature similar to the Lanczos algorithm in Ref. 2. However, it is a more general case because the algorithm includes the output measurement equation. It is also shown that the Krylov model reduction method has an association with the well-known parameter-matching method for general linear systems.[5,6] The reduced-order model obtained by the Krylov model reduction algorithm proposed here is shown to match a set of parameters called low-frequency moments. For a general linear time-invariant system described by

$$\dot{z} = Az + Bu \qquad z \in R^n,\ u \in R^l \tag{1a}$$

$$y = Cz \qquad y \in R^m \tag{1b}$$

the low-frequency moments are defined by $CA^{-i}B$, $i = 1, 2, \ldots$, which are the coefficient matrices in the Taylor series expansion of the system transfer function. One advantage of using Krylov reduced models for structural control design is that the Krylov formulation can eliminate the control and observation spillover terms while leaving only the dynamic spillover terms to be considered.[4]

In this Note, first a theorem is used to show how the Krylov vectors can form a basis to produce a reduced model with moment-matching property. Then, based on the theorem, an efficient algorithm to generate Krylov vectors is developed.

Undamped Structural Dynamics Systems

An undamped structural dynamics system can be described by the input-output equations

$$M\ddot{x} + Kx = Pu \tag{2a}$$

$$y = Vx + W\dot{x} \tag{2b}$$

where $x \in R^n$ is the displacement vector; $u \in R^l$ is the input force vector; $y \in R^m$ is the output measurement vector; M and K are the system mass and stiffness matrices, respectively; P is the force distribution matrix; and V and W are the displacement and velocity sensor distribution matrices, respectively. In most practical cases, it may be assumed that l and m are much smaller than n.

Applying the Fourier transform to Eq. (2a) yields the frequency response solution $X(\omega) = (K - \omega^2 M)^{-1} PU(\omega)$, with $X(\omega)$ and $U(\omega)$ the Fourier transforms of x and u. If the system is assumed to have no rigid-body motion, then a Taylor series expansion of the frequency response around $\omega = 0$ is possible. Thus,

$$X(\omega) = (I - \omega^2 K^{-1}M)^{-1} K^{-1} PU(\omega) = \sum_{i=0}^{\infty} \omega^{2i} (K^{-1}M)^i K^{-1} PU(\omega) \tag{3}$$

Combining Eq. (2b) and Eq. (3), the system output frequency response can be expressed as

$$Y(\omega) = \sum_{i=0}^{\infty} [V(K^{-1}M)^i K^{-1}P + j\omega W(K^{-1}M)^i K^{-1}P]\omega^{2i} U(\omega) \tag{4}$$

In these expressions, $V(K^{-1}M)^i K^{-1}P$ and $W(K^{-1}M)^i K^{-1}P$ play roles similar to that of low-frequency moments in the first-order state-space formulation. Therefore, we have the following definition.

Definition 1. The low-frequency moments of an undamped structural dynamics system described by Eqs. (2) are

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defined by $V(K^{-1}M)^iK^{-1}P$ and $W(K^{-1}M)^iK^{-1}P$, for $i = 0, 1, 2, \ldots$.

Model reduction of structural dynamics systems is usually obtained by using the Rayleigh-Ritz method. The reduced system equation is

$$\bar{M}\ddot{\bar{x}} + \bar{K}\bar{x} = \bar{P}u \qquad \bar{x} \in R^r (r < n) \tag{5a}$$

$$y = \bar{V}\bar{x} + \bar{W}\dot{\bar{x}} \tag{5b}$$

with $x = L\bar{x}$ and $\bar{M} = L^TML$, $\bar{K} = L^TKL$, $\bar{P} = L^TP$, $\bar{V} = VL$, and $\bar{W} = WL$. The projection matrix L can be chosen arbitrarily. Here, we choose L to be the Krylov subspace in order to provide the moment-matching property.

Theorem 1. If span $\{L\}$ = span $\{L_P\, L_V\, L_W\}$ with

$$L_P = [K^{-1}P, (K^{-1}M)K^{-1}P, \ldots, (K^{-1}M)^pK^{-1}P]$$

$$L_V = [K^{-1}V^T, (K^{-1}M)K^{-1}V^T, \ldots, (K^{-1}M)^qK^{-1}V^T]$$

$$L_W = [K^{-1}W^T, (K^{-1}M)K^{-1}W^T, \ldots, (K^{-1}M)^sK^{-1}W^T]$$

for $p, q, s \geq 0$, then the reduced system matches the low-frequency moments $V(K^{-1}M)^iK^{-1}P$ for $i = 0, 1, 2, \ldots, p+q+1$ and $W(K^{-1}M)^iK^{-1}P$, for $i = 0, 1, 2, \ldots, p+s+1$.

Proof. The proof is recursive. First we have the property: if a vector v is such that $K^{-1}v$ is contained in L, i.e., $K^{-1}v = L\alpha$, then $L(L^TKL)^{-1}L^Tv = L(L^TKL)^{-1}L^TK(K^{-1}v) = L(L^TKL)^{-1}L^TKL\alpha = L\alpha = K^{-1}v$. By using this property and the fact that $(K^{-1}M)^iK^{-1}P$, $i = 0, 1, \ldots, p$, $(K^{-1}M)^iK^{-1}V^T$, $i = 0, 1, \ldots, q$, and $(K^{-1}M)^iK^{-1}W^T$, $i = 0, 1, \ldots, s$, are contained in L, it can be shown that

$$L[(L^TKL)^{-1}(L^TML)]^i(L^TKL)^{-1}L^TP = (K^{-1}M)^iK^{-1}P, \qquad i = 0, 1, \ldots, p$$

$$VL[(L^TKL)^{-1}(L^TML)]^i(L^TKL)^{-1}L^T = V(K^{-1}M)^iK^{-1}, \qquad i = 0, 1, \ldots, q$$

$$WL[(L^TKL)^{-1}(L^TML)]^i(L^TKL)^{-1}L^T = W(K^{-1}M)^iK^{-1}, \qquad i = 0, 1, \ldots, s$$

Therefore,

$$\bar{V}(\bar{K}^{-1}\bar{M})^i\bar{K}^{-1}\bar{P} = VL[(L^TKL)^{-1}(L^TML)]^i(L^TKL)^{-1}L^TP = V(K^{-1}M)^iK^{-1}P \qquad \text{for } i = 0, 1, \ldots, p+q+1$$

$$\bar{W}(\bar{K}^{-1}\bar{M})^i\bar{K}^{-1}\bar{P} = WL[(L^TKL)^{-1}(L^TML)]^i(L^TKL)^{-1}L^TP = W(K^{-1}M)^iK^{-1}P \qquad \text{for } i = 0, 1, \ldots, p+s+1$$

■

The L_P matrix in the theorem is the *generalized controllability matrix,* and the L_V and L_W matrices are the *generalized observability matrices* of the dynamic system described by Eqs. (2). The vectors contained in L_P are frequently referred to in the literature as *Krylov vectors,* or *Krylov modes,* which can be generated by a simple recurrence formula

$$Q_1 = K^{-1}P$$

$$Q_{i+1} = K^{-1}MQ_i$$

Krylov modes can be considered to be static modes and they have a clear physical interpretation.[7] The first vector block $K^{-1}P$ is system's static deflection due to the force distribution P. The vector block Q_{i+1} can be interpreted as the static deflection produced by the inertia forces associated with the deflection Q_i. If only the dynamic response simulation is of interest, we usually choose $L = L_P$.[1-3] In this case, the reduced model matches $p+1$ low-frequency moments. As to the vectors in L_V and L_W, a physical interpretation such as the static deflection due to the sensor distribution may be inadequate. However, from an input-output point of view, L_V, L_W, and L_P are equally applicable as far as parameter matching of the reduced-order model is concerned.

Krylov Model Reduction Algorithm

Based on Theorem 1, the following algorithm may be used to generate a Krylov basis that can produce a reduced-order model with the stated parameter-matching property.

Krylov/Lanczos Algorithm

1) Starting vector:
   a) $Q_0 = 0$
   b) $R_0 = K^{-1}\tilde{P}$
      $\tilde{P}$ = linearly independent portion of $[P\ V^T\ W^T]$
   c) $R_0^TKR_0 = U_0\Sigma_0U_0^T$ (singular-value decomposition)
   d) $Q_1 = R_0U_0\Sigma_0^{-1/2}$ (normalization)
2) For $j = 1, 2, \ldots, k-1$, repeat:
   e) $\bar{R}_j = K^{-1}MQ_j$
   f) $R_j = \bar{R}_j - Q_jA_j - Q_{j-1}B_j$ (orthogonalization)
      $A_j = Q_j^TK\bar{R}_j$, $B_j = U_{j-1}\Sigma_{j-1}^{1/2}$
   g) $R_j^TKR_j = U_j\Sigma_jU_j^T$ (singular-value decomposition)
   h) $Q_{j+1} = R_jB_{j+1}^{-T} = R_jU_j\Sigma_j^{-1/2}$ (normalization)
   end
3) Form the k-block projection matrix $L = [Q_1\ Q_2 \ldots Q_k]$.

This algorithm is a modification of the Lanczos algorithm given in Ref. 2. It is a Krylov algorithm because the L matrix is generated by a Krylov recurrence formula (step e). It is a Lanczos algorithm because the orthogonalization scheme is a three-term recursion scheme (step f). Although the three-term recursion scheme is a special feature of the Lanczos algorithm, in practice, complete reorthogonalization or selective reorthogonalization is necessary to prevent the loss of orthogonality.[8-10] The L matrix generated by the algorithm spans the same subspace as the L matrix in Theorem 1. Therefore, according to Theorem 1, if the projection matrix L generated by the algorithm is employed to perform model reduction, then the reduced-order model matches the low-frequency moments $V(K^{-1}M)^iK^{-1}P$ and $W(K^{-1}M)^iK^{-1}P$, for $i = 0, 1, 2, \ldots, 2k-1$. It can also be shown that the reduced-order model approximates the lower natural frequencies of the full-order model.

One interesting feature of the transformed system equation in Krylov coordinates is that it has a special form. Because of the choice of starting vectors, K orthogonalization, and three-term recurrence, the transformed system equation has a mass matrix in block-tridiagonal form, a stiffness matrix equal to the identity matrix, and force distribution and measurement distribution matrices with nonzero elements only in the first block. The transformed system equation has the form

$$\begin{bmatrix} \times & \times & & & \\ \times & \times & \times & & \\ & \times & \times & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot \\ & & & & \cdot & \cdot & \times \\ & & & & & \times & \times \end{bmatrix} \ddot{\bar{x}} + \bar{x} = \begin{Bmatrix} \times \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{Bmatrix} u \tag{6}$$

$$y = [\times\ 0\ 0 \cdots 0]\bar{x} + [\times\ 0\ 0 \cdots 0]\dot{\bar{x}}$$

where $\times$ denotes the location of nonzero elements. This special form reflects the structure of a tandem system (Fig. 1), in which only subsystem S_1 is directly controlled and measured, while the remaining subsystems, S_i, $i = 2, 3, \ldots$, are excited through chained dynamic coupling. In control applications, as depicted in Ref. 4, this tandem structure of the dynamic equa-

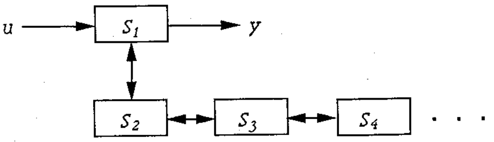

Fig. 1 Structure of a tandem system.

tion eliminates the control and observation spillovers but has dynamic spillover. The trailing zeros in the force distribution and measurement distribution matrices eliminate the control and observation spillovers. Dynamic spillover is due to the nonzero super- and subdiagonal submatrices in the mass matrix. For dynamic response simulation, the block-tridiagonal form can lead to an efficient time-step solution and can save storage space.[2]

Summary

A Krylov model reduction algorithm for an undamped structural dynamics system is presented. The reduced-order model obtained matches low-frequency moments. The transformed system equation in Krylov coordinates reflects the structure of a tandem system.

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Optimal Feedback Gains for Three-Dimensional Large Angle Slewing of Spacecraft

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Introduction

THREE-DIMENSIONAL, large angle slewing or rotational maneuvers occur during retargeting of spacecraft or spacecraft appendages. This is a nonlinear control problem that has received considerable attention in the recent literature (see, e.g., Refs. 1–5). In a representative work, Junkins and Turner[1] consider various methods of optimal control of slewing of rigid and flexible bodies and discuss numerical techniques of solving the associated two-point boundary value problem to obtain open-loop control profiles. A simpler approach is taken by Wie and Barba,[5] who propose feedback control in terms of Euler parameters and angular velocity components and show the effectiveness of such control. Because the dynamics are strongly nonlinear, selection of the feedback gains in this approach is based on good engineering judgment and verified by simulation. This paper provides a systematic procedure for determining these feedback control gains based on optimal control theory[7,8] and an application of a state-of-the-art optimization technique.[9] The method is applied to a problem in which a rigid body is simultaneously rotated in three axes to capture a desired orientation using feedback control torques that have saturation constraints.

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Statement of the Problem

The problem concerns reorienting a rigid body from its current orientation at rest to a desired orientation, also at rest, in a given time in an optimal manner that is to be defined. Let x_1, x_2, x_3, and x_4 denote the four Euler parameters describing the current orientation of a rigid body undergoing large three-dimensional rotation. Kinematical differential equations for Euler parameters are given by[6]

$$\dot{x}_1 = 0.5\ (x_5 x_4 - x_6 x_3 + x_7 x_2) \tag{1a}$$

$$\dot{x}_2 = 0.5\ (x_5 x_3 + x_6 x_4 - x_7 x_1) \tag{1b}$$

$$\dot{x}_3 = 0.5\ (-x_5 x_2 - x_6 x_1 + x_7 x_4) \tag{1c}$$

$$\dot{x}_4 = -0.5\ (x_5 x_1 + x_6 x_2 + x_7 x_3) \tag{1d}$$

where x_5, x_6, and x_7 are the angular velocity components along the body axes. The dynamical equations are given by the following Euler equations:

$$\dot{x}_5 = [T_1 - (I_3 - I_2)x_6 x_7]/I_1 \tag{2a}$$

$$\dot{x}_6 = [T_2 - (I_1 - I_3)x_5 x_7]/I_2 \tag{2b}$$

$$\dot{x}_7 = [T_3 - (I_2 - I_1)x_5 x_6]/I_3 \tag{2c}$$

where T_i and I_i are the ith body axis component of control torque and the centroidal principal moment of inertia for the body, respectively. Feedback control is proposed in the manner of Ref. 5, but with the extra constraint of torque saturation as follows:

$$T_1 = -k_1 e_1 - k_2 x_5 \quad \text{if} \quad |T_1| < T_s$$
$$= T_s \operatorname{sgn}(-k_1 e_1 - k_2 x_5) \qquad \text{otherwise} \tag{3a}$$

$$T_2 = -k_3 e_2 - k_4 x_6 \quad \text{if} \quad |T_2| < T_s$$
$$= T_s \operatorname{sgn}(-k_3 e_2 - k_4 x_6) \qquad \text{otherwise} \tag{3b}$$

$$T_3 = -k_5 e_3 - k_6 x_7 \ \text{if}\ |T_3| < T_s$$
$$= T_s \operatorname{sgn}(-k_5 e_3 - k_6 x_7) \qquad \text{otherwise} \tag{3c}$$

where e_1, e_2, and e_3 are the components of the Euler vector for error in orientation given by the Euler parameters representing the single-axis rotation that it takes to go from the current attitude to the reference attitude. These are computed according to the rules of quaternion algebra[6]

$$e_1 = r_4 x_1 + r_3 x_2 - r_2 x_3 - r_1 x_4 \tag{4a}$$

$$e_2 = -r_3 x_1 + r_4 x_2 + r_1 x_3 - r_2 x_4 \tag{4b}$$